Radioactivity & Decay Law

1 Mark Questions

1. Why is it found experimentally difficult to detect neutrinos in nuclear β-decay? **[All India 2014]**

Ans.Neutrinos are difficult to detect because they are massless, have no charge and do not interact with nucleons.

2.Define the activity of a given radioactive substance. Write its SI unit. [All India 2013]

Ans.The activity of a sample is defined as the rate of disintegration taking place in the sample of radioactive substance.

SI unit of activity is Becquerel (Bq).

1 Bq =1 disintegration/second

3.Write any two characteristic properties of nuclear force. [All India 2011]

Ans. Two characteristics of nuclear force are given as below:

(i) These are short range forces.

(ii) These are strong force of attractive nature

4.How is the mean life of a radioactive sample related to its half-life? [Foreign 2011]

Ans.

Mean life, $t_m = \frac{1}{\lambda}$, where λ is decay constant.

But half-life, $T_{1/2} = \frac{\ln 2}{\lambda}$

$$\Rightarrow \quad \lambda = \frac{\ln 2}{T_{1/2}}$$

$$\Rightarrow \quad t_m = \frac{T_{1/2}}{\ln 2} = \frac{T_{1/2}}{0.693}$$

$$t_m = 1.443\, T_{1/2}$$

5.How is the radius of a nucleus related to its mass number? [All India 2011 c]

Ans.

Radius of nucleus, $R = R_0 A^{1/3}$

where, R = radius of nucleus

A = mass number

$R_0 = 1.2 \text{ fm} = 1.2 \times 10^{-15} \text{m}$

6. A nucleus undergoes β-decay. How does its

(i) mass number

(ii) atomic number change? **[Delhi 2011C]**

Ans.

During β-decay,

(i) no change in mass number.

(ii) atomic number increases by 1.

7. A nucleus $^{238}_{92}U$ undergoes α-decay and transforms to thorium. What is

(i) the mass number and

(ii) atomic number of the nucleus produced? **[Delhi 2011C]**

Ans.

In α-decay, the mass number of parent nucleus decreases by 4 units and atomic number decreases by 2 units.

$$\therefore \quad {}^{238}U_{92} \xrightarrow{\alpha\text{-decay}} {}^{234}Th_{90}$$

(i) Mass number of the nucleus produced $= 234$ **(1/2)**

(ii) Atomic number of nucleus produced $= 90$ **(1/2)**

8.Two nuclei have mass numbers in the ratio 1: 8. What is the ratio of their nuclear radii? [All India 2009]

Ans.

Radius of nucleus, $R = R_0 A^{1/3}$

where, $R_0 =$ constant, $A =$ mass number

$$\therefore \quad \frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{1}{8}\right)^{1/3} = \frac{1}{2}$$

$$R_1 : R_2 = 1:2$$

9.Two nuclei have mass numbers in the ratio 8:125. What is the ratio of their nuclear radii? [All India 2009]

Ans. Refer to ans. 8. (Ans. 2 :5)

10.Two nuclei have mass numbers in the ratio 27 :125. What is the ratio of their nuclear radii? [All India 2009]

Ans. Refer to ans. 8. (Ans. 3:5)

11.Two nuclei have mass numbers in the ratio 1:2. What is the ratio of their nuclear densities? [Delhi 2009]

Ans. Nuclear density is independent of mass number.

12.Assuming the nuclei to be spherical in shape, how does the surface area of a nucleus of mass number A1 compare with that of a nucleus of mass number A_2 ? [All India 2008 C]

Ans.

Radius of nucleus, $R = R_0 A^{1/3}$

where, R_0 = constant

$\therefore$ Surface area, $S = 4\pi R^2$

$= 4\pi (R_0 A^{1/3})^2 = 4\pi \; R_0^2 A^{2/3}$

$\therefore$ Ratio of surface areas, $\dfrac{S_1}{S_2} = \left(\dfrac{A_1}{A_2}\right)^{2/3}$

13.Out of the two characteristics, the mass number A and the atomic number Z of a nucleus, which one does not change during nuclear decay? [All India 2008 C]

Ans.

The mass number A of a nucleus does not change during nuclear β-decay. **(1)**

2 Marks Questions

14. In both β^- and β^+- decay process, the mass number of nucleus remains the same, whereas the atomic number Z increases by one in β^--decay and decrease by one in β^+-decay. Explain giving reasons. **[Foreign 2014]**

Ans.

In β^--decay, a β-particle of zero mass and -1 charge is emitted.The decay process is shown as below:

$${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}\beta + \bar{\nu}$$

Since, the mass of β^--particle is negligibly small and the atomic number increases by 1 due to the loss of 1 negative charge. **(1)**

Similarly, for a β^+-decay, a β-particle of negligibly small and $+1$ charge is emitted. The decay process is shown as below:

$${}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + {}^{0}_{+1}\beta + \nu$$

The mass number remains the same but here the atomic number decreases by 1 due to the loss of 1 positive charge. **(1)**

15. In a given sample, two radio isotopes A and B are initially present in the ratio of 1 : 4. The half-lives of A and B are 100 yr and 50 yr, respectively. Find the time after which the amounts of A and B become equal. **[HOTS; Foreign 2012]**

Ans.

In these types of questions, we have to keep in mind the exponential decay.

Let N_A be the concentration of A after time t_A and N_B be the concentration of B after time t_B. From radioactive disintegration equation,

$$N_A = N_0 e^{-\lambda_A t_A}$$

$$N_B = 4N_0 e^{-\lambda_B t_B} \quad [\text{As}, N_{0B} = 4N_{0A}]$$

Now, half-life of A is 100 yr and B is 50 yr.

So, $\lambda_A = \frac{\ln 2}{100}$ and $\lambda_B = \frac{\ln 2}{50}$

Dividing, we get

$$\frac{\lambda_A}{\lambda_B} = \frac{1}{2} \text{ or } \lambda_B = 2\lambda_A$$ **(1)**

Let after t years, $N_A = N_B$

So, $\frac{N_A}{N_B} = \frac{e^{-\lambda_A t}}{4e^{-\lambda_B t}}$ $N_A = N_B$

$\Rightarrow \quad 4e^{-\lambda_B t} = e^{-\lambda_A t}$

$\Rightarrow \quad 4 = e^{-(\lambda_A - \lambda_B)t}$

$\ln 4 = -(+\lambda_A - 2\lambda_A)t \quad [\because \lambda_B = 2\lambda_A]$

$\ln 4 = \lambda_A t$

$$t = \frac{\ln 4}{\ln 2} \times 100 = 200 \text{ yr} \quad \left[\because \lambda_A = \frac{\ln 2}{100}\right]$$ **(1)**

16.How the size of a nucleus is experimentally determined? Write the relation between the radius and mass number of the nucleus. Show that the density of nucleus is independent of its mass number.

[Delhi, 2012,2011 C]

Ans.

The size of nucleus will increase with the increase of mass number.

The size of the nucleus is experimentally determined using Rutherford's α-scattering experiment and the distance of closest approach and impact parameter.

The relation between radius and mass number of nucleus is

$$R = R_0 A^{1/3}, \text{ where } R_0 = 1.2 \text{ fm}$$

where, A = mass number, R = radius of nucleus **(1)**

Nuclear density,

$$\rho = \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}} = \frac{mA}{\frac{4}{3}\pi (R_0 A^{1/3})^3}$$

where, m = mass of each nucleon

$$\rho = \frac{mA}{\frac{4}{3}\pi R_0^3 A} \Rightarrow \rho = \frac{m}{\frac{4}{3}\pi R_0^3}$$

From the above formula, it is clear that ρ does not depend on the mass number. **(1)**

17. In β-decay, the experimental detection of neutrino is found to be difficult. **[Delhi, 2012, 2011C]**

Ans.

18. A radioactive nucleus A undergoes a series of decays according to the following scheme

$$A \xrightarrow{\alpha} A_1 \xrightarrow{\beta} A_2 \xrightarrow{\alpha} A_3 \xrightarrow{\gamma} A_4$$

The mass number and atomic number of A_4 are 172 and 69, respectively. What are these numbers for A? **[Delhi 2009]**

Ans.

In α-decay, the atomic number decreases by 2 units and mass number decreases by 4 units. In β-decay, the atomic number increases by 1 unit but mass number does not change. In γ-decay, there is no change in atomic number and mass number. **(1)**

Let the mass number and atomic number of A be X and Y, respectively.

So, ${}_Y A^X \xrightarrow{\alpha} {}_{Y-2}A_1^{X-4} \xrightarrow{\beta} {}_{Y-2+1}A_2^{X-4}$

or ${}_{Y-1}A_2^{X-4} \xrightarrow{\alpha} {}_{Y-1-2}A_3^{X-4-4}$

or ${}_{Y-3}A_3^{X-8} \xrightarrow{\gamma} {}_{Y-3}A_4^{X-8}$

According to the question, the mass number and atomic number of A_4 are 172 and 69.

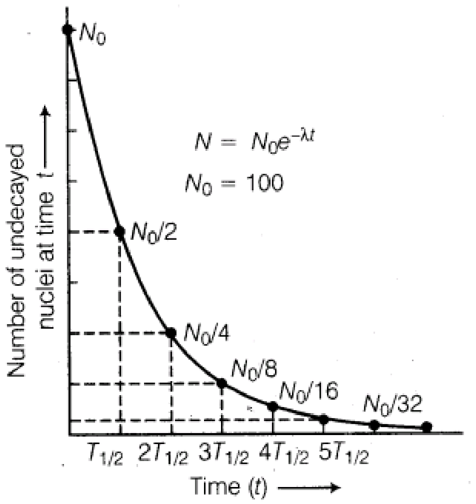
$\therefore \quad X - 8 = 172 \Rightarrow X = 172 + 8 = 180$

$Y - 3 = 69 \Rightarrow Y = 72$ **(1)**

19.Draw a plot representing the law of radioactive decay . Define the activity of a sample of a radioactive nucleus. Write its SI unit. [Foreign 2008]

Ans.

The curve representing the law of radioactive decay is shown as below:



Decay curve for a radioactive element **(1)**

The rate or activity of a sample is defined as the rate of disintegration taking place in the sample of radioactive substance.

The SI unit of activity is Becquerel (Bq).

1 Bq = 1 disintegration /second **(1)**

20. A radio nuclide sample has N_0 nuclei at $t = 0$. Its number of undecayed nuclei get reduced to $\frac{N_0}{e}$ at $t = \tau$. What does the term τ stand for? Write in terms of τ the time interval T in which half of the original number of nuclei of this radio nuclide would have got decayed.

[Delhi 2008C]

Ans.

(*i*) The term τ stands for mean life.

(*ii*) The required relation is $\tau = 1.44T$.

i.e. mean life of radioactive sample

$= 1.44 \times$ half-life

3 Marks Questions

21. (*i*) Deduce the expression, $N = N_0 e^{\lambda t}$ for the law of radioactive decay.

(*ii*) (*a*) Write symbolically the process expressing the β^+- decay of $^{22}_{11}\text{Na}$. Also, write the basic nuclear process underlying this decay.

(*iii*) Is the nucleus formed in the decay of the nucleus $^{22}_{11}\text{Na}$ is isotope or isobar?

[Delhi 2014]

Ans.

(*i*) Rutherford and Soddy made experimental study of the radioactive decay of various radioactive materials and found that the decay of all radioactive materials is governed by the same general law.

According to this law, the rate of decay of radioactive atoms at any instant is proportional to the number of atoms present at that instant.

Let N be the number of atoms present in a radioactive substance at any instant t. Let dN be the number of atoms that disintegrate in a short interval dt. Then, the rate of disintegration $-dN/dt$ is proportional to N, i.e.

$$-\frac{dN}{dt} = \lambda N$$

where, λ is a constant for the given substance and is called **decay constant** (or disintegration constant or radioactive constant or transformation constant). For a given element, the value of λ is constant but for different elements it is different. From the above equation, we have

$$\frac{dN}{N} = -\lambda dt \quad \textbf{(1)}$$

On integrating both sides, we get

$$\log_e N = -\lambda t + C \quad ..(i)$$

where, C is the integration constant. To determine C, we apply the initial conditions. Suppose, there were N_0 atoms in the beginning, i.e., $N = N_0$ at $t = 0$.

Then, $\log_e N_0 = C$

Substituting this value of C in the above Eq.(i), we have

$$\log_e N = -\lambda t + \log_e N_0$$

$$\log_e N - \log_e N_0 = -\lambda t \text{ or } \log_e \frac{N}{N_0} = -\lambda t$$

$$\frac{N}{N_0} = e^{-\lambda t} \Rightarrow N = N_0 e^{-\lambda t} \quad ...(ii)$$

Here, N_0 and N are numbers of atoms in a radioactive substance at time $t = 0$ and after time t respectively. **(1)**

(*ii*) (a) $_{11}Na^{22} \xrightarrow{\beta^+\text{-decay}} {}_{10}Ne^{22} + {}^{0}_{1}e + \nu + Q$

The basic nuclear process is given by

$${}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + {}^{0}_{+1}e + \nu$$

For β^+-decay, there is a conversion of a proton into neutron to emit positron (positive electron).

$$p \rightarrow n + e^+ + \nu \quad \textbf{(1/2)}$$

(*iii*) The nucleus formed in the decay is an isobar. **(1/2)**

22. (*i*) Define the terms (*a*) half-life (*b*) average life. Find out the relationship with the decay constant (λ).

(*ii*) A radioactive nucleus has a decay constant $\lambda = 0.3465\ (\text{day})^{-1}$. How long would it take the nucleus of decay to 75% of its initial amount? **[Foreign 2014]**

Ans.

(i) (a) **Half-life** Half-life of a radioactive element is defined as the time during which half the number of atoms present initially in the sample of the element decay or it is the time during which number of atoms left undecayed in the sample is half the total number of atoms present in the sample. It is represented by $T_{1/2}$.

From the equation $N = N_0 e^{-\lambda t}$,

At half-life, $t = T_{1/2}$, $N = \frac{N_0}{2}$

$$\therefore \frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}} \Rightarrow \frac{1}{2} = e^{-\lambda T_{1/2}}$$

$$\Rightarrow e^{\lambda T_{1/2}} = 2$$

On taking log on both sides, we get

$$\lambda T_{1/2} = \log_e 2$$

$$T_{1/2} = \frac{\log_e 2}{\lambda} = \frac{\log_{10} 2 \times 2.303}{\lambda}$$

$$= \frac{0.3010 \times 2.303}{\lambda}$$

After n half-life, the the number of atoms left undecayed is given by $N = N_0 \left(\frac{1}{2}\right)^n$

$$T_{1/2} = \frac{0.6932}{\lambda}$$ **(1)**

(b) **Average Life** Average life of a radioactive element can be obtained by calculating the total life time of all atoms of the element and dividing it by the total number of atoms present initially in the sample of the element.

Average life or mean life of radioactive element is

$$\tau = \frac{\text{Total life time of all atoms}}{\text{Total number of atoms}}$$

$$\tau = \int_0^{N_0} \frac{t dN}{N_0} = \int_\infty^0 \frac{-\lambda N_0 e^{-\lambda t} dt \times t}{N_0}$$

[When $N = N_0, t = 0$ and when $N = 0, t = \infty$]

[$\because dN = -\lambda (N_0 e^{-\lambda t})\, dt$]

$$= \lambda \int_0^\infty t e^{-\lambda t} dt = \lambda \left[\left\{ t \frac{e^{-\lambda t}}{-\lambda} \right\}_0^\infty - \int_0^\infty \frac{e^{-\lambda t}}{-\lambda} dt \right]$$

$$= \lambda \left(0 + \frac{1}{\lambda} \int_0^\infty e^{-\lambda t} dt \right)$$

$$= \int_0^\infty e^{-\lambda t} dt = \left[\frac{e^{-\lambda t}}{-\lambda} \right]_0^\infty = 0 - \frac{1}{-\lambda} = \frac{1}{\lambda}$$

$$\tau = \frac{1}{\lambda} = \frac{1}{0.6931 / T_{1/2}}$$

$$\tau = 1.44\, T_{1/2}$$ **(1)**

(*ii*) Given, $\lambda = 0.3465\ (\text{day})^{-1}$

According to the radioactive decay law, we have

$$R = R_0 e^{-\lambda t}$$

$$\Rightarrow \frac{R_0 \times 75}{100} = R_0 e^{-0.3465t} \Rightarrow \frac{3}{4} = e^{-0.3465t}$$

$$\Rightarrow \quad t = 0.830\ \text{s}$$ **(1)**

23. (*i*) Define the term activity of a sample of radioactive nucleus.Write its SI unit.

(*ii*) The half-life of $^{238}_{92}\text{U}$ undergoing α-decay is 4.5×10^9 yr. Determine the activity of 10g sample of $^{238}_{92}\text{U}$. Given that 1g of $^{238}_{92}\text{U}$ contains 25.3×10^{20} atoms. **[All India 2014 C]**

Ans.

(*i*) The activity of a sample of radioactive nucleus equals to its decay rate (or number of nuclei decaying per unit time). Its SI unit is disintegrations or Becquerel. $\left(1\frac{1}{2}\right)$

(*ii*) Given,

Number of atoms in 1 g of $^{238}_{92}\text{U}$ is 25.3×10^{20} atoms.

From the radioactive decay,

$$\frac{dN}{dt} \propto N \Rightarrow R \propto N \qquad \left[\because \frac{dN}{dt} = R\right]$$

$$R = \lambda N = \frac{\log_e 2 \times 25.3 \times 10^{20} \times 10}{4.5 \times 10^9}$$

$$= \frac{0.6931 \times 25.3 \times 10^{21}}{4.5 \times 10^9 \times 365 \times 24 \times 60 \times 60}$$

$$= 1.24 \times 10^5 \text{dps}$$ $\left(1\frac{1}{2}\right)$

24. (*i*) The number of nuclei of a given radioactive sample at time $t = 0$ and $t = T$ are N_0 and N_0/n, respectively. Obtain an expression for the half-life ($T_{1/2}$) of the nucleus in terms of n and T.

(*ii*) Write the basic nuclear process underlying β-decay of a given radioactive nucleus. **[Delhi 2013C]**

Ans.

(i) According to the law of radioactive decay,

$$N = N_0 e^{-\lambda t}$$

$$N = \frac{N_0}{n} \text{ and } t = T$$

$$\therefore \quad \frac{N_0}{n} = N_0 e^{-\lambda T}$$

$$\Rightarrow \quad n = e^{\lambda T}$$

$$\Rightarrow \quad \lambda = \frac{\log n}{T}$$

$\therefore$ Half-life, $T_{1/2} = \frac{0.6931}{\lambda} = \frac{0.693T}{\log n}$ $\left(1\frac{1}{2}\right)$

(ii) In β^-- decay process, a nucleus emits a negative charge. A neutron is converted to a proton causing the nuclide's atomic number to increase by one but the atomic mass remains the same.

e.g. ${}^{14}_{6}C \longrightarrow {}^{14}_{7}N + e^- + \nu$ $\left(1\frac{1}{2}\right)$

25. State the law of radioactive decay. Plot a graph showing the number N of undecayed nuclei as a function of time t-for a given radioactive sample having half-life $T_{1/2}$.
Depict in the plot, the number of undecayed nuclei at
(i) $t = 3\,T_{1/2}$ (ii) $t = 5\,T_{1/2}$. **[Delhi 2011]**

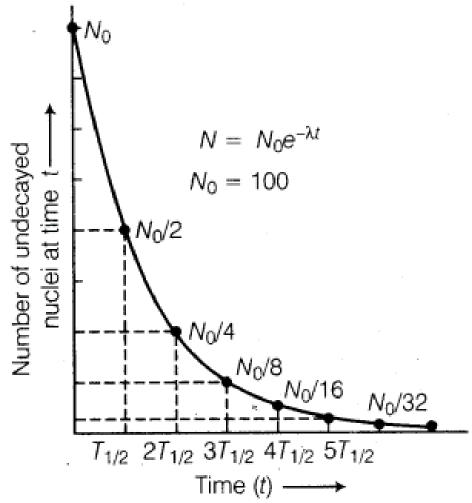
Ans.

Law of Radioactive Decay The rate of disintegration of radioactive sample at any instant is directly proportional to the number of undisintegrated nuclei present in the sample at that instant, i.e.

$$\frac{dN}{dt} \propto N \qquad \frac{dN}{dt} = -\lambda N$$ **(1)**

where, N = number of undisintegrated nuclei present in the sample at any instant t and $\frac{dN}{dt}$ is rate of disintegration. **(1)**

The curve representing the law of radioactive decay is shown as below:



Decay curve for a radioactive element **(1)**

The rate or activity of a sample is defined as the rate of disintegration taking place in the sample of radioactive substance.

The SI unit of activity is Becquerel (Bq).

1 Bq = 1 disintegration /second **(1)**

26. (*i*) Define activity of a radioactive material and write its SI unit.

(*ii*) Plot a graph showing variation of activity of a given radioactive sample with time.

(*iii*) The sequence of stepwise decay of a radioactive nucleus is

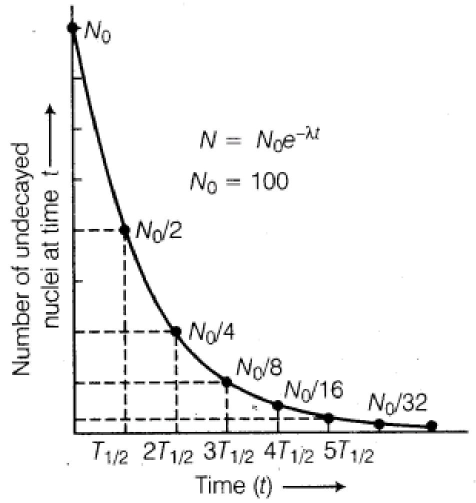
$$D \xrightarrow{\beta\text{-particle}} D_1 \xrightarrow{\alpha\text{-particle}} D_2$$

If the atomic number and mass number of D_2 are 71 and 176 respectively, what are their corresponding values for

[HOTS; Delhi 2010]

Ans.(i)

The curve representing the law of radioactive decay is shown as below:



Decay curve for a radioactive element **(1)**

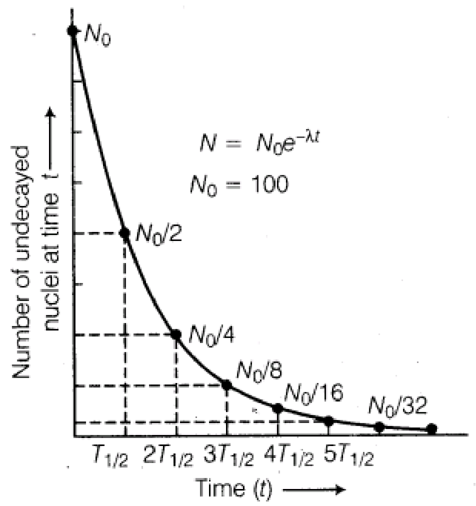
The rate or activity of a sample is defined as the rate of disintegration taking place in the sample of radioactive substance.

The SI unit of activity is Becquerel (Bq).

1 Bq = 1 disintegration /second **(1)**

(ii)

The curve representing the law of radioactive decay is shown as below:



Decay curve for a radioactive element **(1)**

The rate or activity of a sample is defined as the rate of disintegration taking place in the sample of radioactive substance.

The SI unit of activity is Becquerel (Bq).

1 Bq = 1 disintegration /second **(1)**

(iii) In these types of questions, remember the change in atomic number and mass number during the emission of α, β or γ.

In β-decay, the mass number remains same and atomic number increases by 1 unit. In α-decay, the mass number decreases by 4 units and atomic number decreases by 2 units.

Let mass and atomic number of D be A and Z, respectively.

$$\therefore \; {}^{A}_{Z}D \xrightarrow{\beta^- \text{-particle}} {}^{A}_{Z+1}D_1 \xrightarrow{\alpha\text{-particle}} {}^{A-4}_{Z+1-2}D_2 \text{ or } {}^{A-4}_{Z-1}D$$

According to the question, the mass number and atomic number of D_2 are 176 and 71, respectively.

(a) Atomic number of $D = Z - 1 = 71$

$\Rightarrow \quad Z = 72$ **(1/2)**

(b) Mass number of $D = A - 4 = 176$

$\Rightarrow \quad A = 176 + 4 = 180$ **(1/2)**

27. What is the basic mechanism for the emission of β^- and β^+-particles in a nuclide? Give an example by writing explicitly a decay process for β-emission. Is

(i) the energy of the emitted β-particles continuous or discrete?

(ii) the daughter nucleus obtained through β-decay, an isotope or an isobar of the parent nucleus?

[Delhi 2010C]

Ans.

During β^--decay from the nucleus, nucleus undergoes a change in such a way that atomic number increases by one and mass number remains same.

In β^+-decay, the mass number of present radioactive nucleus remains same, whereas atomic number decreases by one. **(1)**

Example of β^--decay

$${}^{32}_{15}P \rightarrow {}^{32}_{16}S + e^- + \overline{\nu}$$

[Half-life = 26 days]

In β-decay, an electron and an antineutrino are created in the following manner.

$$n \rightarrow p + e^- + \overline{\nu}$$

(i) The energy of emitted β-particles is continuous. **(1)**

(ii) As, there is no change in mass number during β-decay. So, the daughter nucleus is isobar of the parent nucleus. **(1)**

28. (*i*) What is meant by half-life of a radioactive element?

(*ii*) The half-life of a radioactive substance is 30 s. Calculate

(*a*) the decay constant and

(*b*) time taken for the sample to decay by 3/4th of the initial value. **[Foreign 2009]**

Ans.

(*i*) (a) **Half-life** Half-life of a radioactive element is defined as the time during which half the number of atoms present initially in the sample of the element decay or it is the time during which number of atoms left undecayed in the sample is half the total number of atoms present in the sample. It is represented by $T_{1/2}$.

From the equation $N = N_0 e^{-\lambda t}$,

At half-life, $t = T_{1/2}$, $N = \frac{N_0}{2}$

$$\therefore \frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}} \Rightarrow \frac{1}{2} = e^{-\lambda T_{1/2}}$$

$$\Rightarrow e^{\lambda T_{1/2}} = 2$$

On taking log on both sides, we get

$$\lambda T_{1/2} = \log_e 2$$

$$T_{1/2} = \frac{\log_e 2}{\lambda} = \frac{\log_{10} 2 \times 2.303}{\lambda}$$

$$= \frac{0.3010 \times 2.303}{\lambda}$$

After n half-life, the the number of atoms left undecayed is given by $N = N_0\left(\frac{1}{2}\right)^n$

$$T_{1/2} = \frac{0.6932}{\lambda}$$ **(1)**

(*b*) **Average Life** Average life of a radioactive element can be obtained by calculating the total life time of all atoms of the element and dividing it by the total number of atoms present initially in the sample of the element.

Average life or mean life of radioactive element is

$$\tau = \frac{\text{Total life time of all atoms}}{\text{Total number of atoms}}$$

$$\tau = \int_0^{N_0} \frac{t dN}{N_0} = \int_{\infty}^{0} \frac{-\lambda N_0 e^{-\lambda t} dt \times t}{N_0}$$

[When $N = N_0, t = 0$ and when $N = 0, t = \infty$]

[$\because dN = -\lambda(N_0 e^{-\lambda t})\, dt$]

$$= \lambda \int_0^\infty t e^{-\lambda t} dt = \lambda \left[\left\{ t \frac{e^{-\lambda t}}{-\lambda} \right\}_0^\infty - \int_0^\infty \frac{e^{-\lambda t}}{-\lambda} dt \right]$$

$$= \lambda \left(0 + \frac{1}{\lambda} \int_0^\infty e^{-\lambda t} dt \right)$$

$$= \int_0^\infty e^{-\lambda t} dt = \left[\frac{e^{-\lambda t}}{-\lambda} \right]_0^\infty = 0 - \frac{1}{-\lambda} = \frac{1}{\lambda}$$

$$\tau = \frac{1}{\lambda} = \frac{1}{0.6931/T_{1/2}}$$

$$\tau = 1.44\, T_{1/2}$$ **(1)**

(*ii*) Given, $\lambda = 0.3465\ (\text{day})^{-1}$

According to the radioactive decay law, we have

$$R = R_0 e^{-\lambda t}$$

$$\Rightarrow \frac{R_0 \times 75}{100} = R_0 e^{-0.3465t} \Rightarrow \frac{3}{4} = e^{-0.3465t}$$

$$\Rightarrow \quad t = 0.830 \text{ s}$$ **(1)**

(*ii*) $T_{1/2} = 30$ s

(a) $\lambda = ?$

$$\because \quad T_{1/2} = \frac{0.693}{\lambda}$$

$$\Rightarrow \quad \lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{30} = 0.0231 \text{ s}^{-1}$$ **(1)**

(*b*) $\because \quad N = N_0 \left(\frac{1}{2}\right)^n$

where, n = number of half-lives

N = number of undisintegrated nuclei present in the sample

N_0 = original number of undisintegrated atom

undisintegrated atom

Here, $N = N_0 - \frac{3}{4} N_0$

$$N = \frac{1}{4} N_0 \Rightarrow N = N_0 \left(\frac{1}{2}\right)^n$$

$$\frac{N_0}{4} = N_0 \left(\frac{1}{2}\right)^n \Rightarrow \left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^n$$

$$\Rightarrow \quad n = 2$$

But number of half-lives

$$2 = \frac{\text{Total time taken}}{30 \text{ s}}$$

Total time taken = 60 s = 1 min **(1)**

29. An observer in a laboratory starts with N_0 nuclei of a radioactive sample and keep on observing the number (N) of left over nuclei at regular intervals of 10 min each. She prepares the following table on the basis of her observation.

Time t (in min)	$\log_e\left(\frac{N_0}{N}\right)$
0	0
10	3.465
20	6.930
30	10.395
40	13.860

Use this data to plot a graph of $\log_e(N_0/N)$ vs time (t) and calculate the

(i) decay constant and

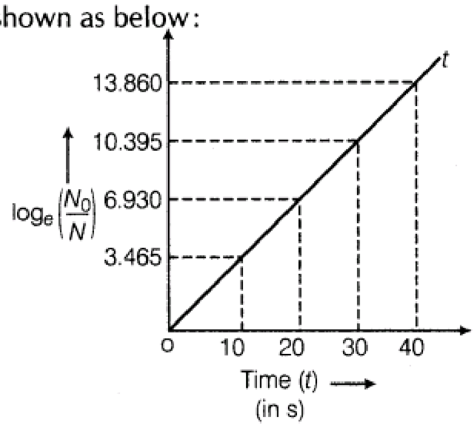
(ii) half-life of the given sample.

[Delhi 2009C]

Ans.

> We have to keep in mind these things while drawing the graph.
>
> The graph will be straight line passing through the origin. The slope of the graph will be constant.

The graph between $\log_e\left(\frac{N_0}{N}\right)$ and time is shown as below:



(1)

(*i*) $\because \quad \log_e\left(\frac{N_0}{N}\right) = \lambda t$

$\Rightarrow$ Slope of $\log_e\left(\frac{N_0}{N}\right)$ *vs* time *t* graph gives decay constant λ.

$\therefore \quad \lambda = \frac{3.465}{10} \text{ s}^{-1}$

[using observation given in table]

$= \frac{6.930}{20}$

$= 0.3465$ **(1)**

(*ii*) $\because$ Half-life, $T_{1/2} = \frac{0.693}{\lambda}$

$T_{1/2} = \frac{0.693}{0.3465} = 2 \text{ s}$

Half-life, $T_{1/2} = 2 \text{ s}$ **(1)**